(1) **Yes, it is possible.** For example: look at the graph here, where $r$ is the starting node:

![BFS Tree 1](image1)

The BFS tree looks like:

![BFS Tree 2](image2)

We observe that: $S = \{(b, c)\}$. Now, $d(b) = 1, d(c) = 1$, and $(b, c) \in S$

(2) **Yes, it is possible.** For example: look at the graph here where $r$ is the starting node:

![BFS Tree 3](image3)

Two BFS trees are possible here. Let us take one of them:

![BFS Tree 4](image4)

We observe that: $S = \{(c, e)\}$. Now, $d(c) = 1, d(e) = 2$, and $(c, e) \in S$

(3) **No, it is not possible.** Let us assume that: a node $a$ is in layer $k$, node $b$ be in layer $k + t$ where $t > 1$ and $(a, b) \in S$. Thus, we know that $(a, b) \in G$. Now, $a$ will be discovered first in level $k$, and all its neighbors will be discovered in level $k + 1$. Since $(a, b) \in G$, $b$ is one of the neighbors of $a$. Thus, $t = 1$, which contradicts our above assumption.

![BFS Tree 5](image5)