Assignment # 1
(Due February 9, 2016)

1. Recall that Kargers Contraction algorithm on an input graph \( G \) repeats the following process: randomly pick an edge in the graph then contract it until the graph becomes a 2-vertex graph \( G_2 \). Then the algorithm outputs all edges in \( G_2 \) as a cut for the original input graph \( G \). Suppose that the input graph \( G \) has \( n \) vertices and is represented by an \( n \times n \) matrix \( M_G \), where \( M_G[i,j] \) is the number of edges between the vertices \( i \) and \( j \).

Give detailed descriptions on how the random edge selections and the edge contractions are implemented based on the matrix \( M_G \) and how the edges in the final graph \( G_2 \) are translated back to the corresponding edges in the original graph \( G \). The running time of Kargers Contraction algorithm should be bounded by \( O(n^2) \) under your implementation.

2. Now suppose that we modify Kargers Contraction algorithm as follows. First we repeat the process of randomly contracting edges until the graph becomes a \( t \)-vertex graph \( G_t \), then we use an \( O(t^3) \)-time deterministic algorithm to find the min-cut of \( G_t \).

What is the time complexity of this modified algorithm, and what is the probability that this modified algorithm produces a min-cut for the original graph? How many times should we repeat this modified algorithm in order to produce a min-cut of the original graph with a probability at least 99%? Note that your answers for the above questions all depend on \( t \).

Discuss what value of \( t \) we should take in order to optimize the time complexity of the above algorithm with a success probability 99%.

3. Henry and Tom play a game by tossing a fair coin. If the coin turns head then Henry gains 1 point and Tom gains 0 point, and if the coin turns tail then Henry gains 0 points while Tom gains 1 point. The game is over when either Henry gets 2 points before Tom gets 3 points (in this case Henry wins) or Tom gets 3 points before Henry gets 2 points (in this case Tom wins).

List all outcomes of the game, and give the probability for each outcome. What is the probability that Henry wins? What is the probability that Tom wins?