1. In how many ways can $n$ balls be put into $n$ numbered boxes so that exactly one box is empty? You should distinguish the cases where the balls are distinguishable and where the balls are indistinguishable.

2. Using the technique of random permutation to develop a randomized algorithm of running time $O(n^2k!)$ that, on a given weighted and directed graph $G$, either returns a $k$-path of the maximum weight in $G$ or reports that no $k$-paths exist in $G$.

3. Using the technique of color-coding to develop a randomized algorithm for the following problem: Given a graph $G$ and an integer $k$, either construct $k$ vertex-disjoint triangles in $G$, or report that no such $k$ triangles exist. What is the time complexity of your algorithm?

Remark. The problems given in Questions 2 and 3 are both NP-hard.