Assignment # 3
(November 11)

1. Metric-TSP. Write detailed algorithm and analysis for the approximation algorithm based on minimum spanning trees for the metric traveling salesman Problem (in which edge weight for any three vertices $x, y,$ and $z$ satisfies $wt(x, y) \leq wt(x, z) + wt(z, y)$). Your algorithm should contain the details of how the traveling salesman tour (i.e., the permutation of the vertices) is constructed, and your analysis should give the details on why the tour is at most twice of the optimal tour.

2. Planar Graph Coloring. Show that the best ratio of a polynomial-time approximation algorithm for the planar graph coloring problem (given a planar graph $G$, color the vertices of $G$ using a minimum number of colors such that no two adjacent vertices get the same color) is $4/3$. You can assume a polynomial-time algorithm that 4-colors any planar graph. Your solution should contain an approximation algorithm of ratio $4/3$ for the problem, and a proof that no polynomial-time approximation algorithm for the problem can have ratio smaller than $4/3$ unless $P = NP$.

3. 3-Dimension Matching. Write a polynomial-time approximation algorithm for the following 3D-MATCHING problem: given a set $S$ of $n$ points in 3-D Euclidean space $E^3$, find a maximum subset $M$ of $S$ such that no two points in $M$ have the same value on any coordinate. Your algorithm should have an approximation ratio not larger than 3. (Remark: a ratio 2 can be achieved with a little more efforts, and the currently best ratio is $1.5 + \epsilon$.)