

References for direct methods for sparse linear systems

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All of the following references appear in our Acta Numerica paper, *A survey of direct methods for sparse linear systems*, by Davis, Rajamanickam, and Sid-Lakhdar, Acta Numerica, vol 25, May 2016, pp. 383-566, with one additional reference: the survey paper itself [138]. A bibtex file for all the references is available as the `survey.bib`, posted at <http://faculty.cse.tamu.edu/davis/publications.html>

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