Problem Set 9

CPSC 411 Analysis of Algorithms

Andreas Klappenecker

The assignment is due Thursday, Dec 03, 2009, before class.

Exercise 1. *Let $G = (V, E)$ be an undirected graph, V' a subset of V. Denote by G_c the complementary graph of G, i.e., G_c has vertex set V and contains precisely the edges that are missing in G. Show that V' is a clique in G if and only if $V - V'$ is a vertex cover of G_c. [Hint: We stated this theorem in class but did not fully prove it.]*

Exercise 2. *Exercise 34.5-5 on page 1017 of our textbook.*

Exercise 3. *The SET COVER problem is the following decision problem: Given a family of sets $S_1, S_2, \dots, S_n$ and a integer k in the range $1 \leq k \leq n$. Does there exist a subfamily of k sets $S_{i_1}, S_{i_2}, \dots, S_{i_k}$ such that*

$$\bigcup_{j=1}^{k} S_{i_j} = \bigcup_{j=1}^{n} S_j?$$

Show that SET COVER is NP-complete. Use a polynomial reduction from VERTEX COVER to SET COVER. [Hint: Reductions do not need to be complicated! Try to re-interpret the VERTEX COVER problem in terms of sets.]

Exercise 4. *Let us construct an undirected graph $G = (V, E)$ as follows. The nodes V are the courses offered by a university during the current semester. There is an edge $\{u, v\}$ in E if and only if the courses u and v have at least one student in common. The decision problem FINALS is: Given the graph G and an integer k, does there exist k or more courses whose finals can be scheduled at the same time? Show that FINALS is NP-complete. [Hint: Clique is NP-complete]*

Exercise 5. *Prove that the set $\mathbf{Z}$ of integers is countable by explicitly giving a bijective function $f : \mathbf{Z} \to \mathbf{N}$.*

Exercise 6. *Let S be the set of infinite binary sequences. Show that the set S is uncountable.*