Problem Set 1
CPSC 411 Analysis of Algorithms
Andreas Klappenecker

The assignment is due next Friday, Sep 9, 2011, before class.

Exercise 1 (10 points). Use the definition of Big-Oh notation to prove that

\[ n \lfloor \log n \rfloor = O(n^2). \]

Here \( \lfloor x \rfloor \) denotes the floor function that yields the largest integer \( \leq x \) as a value.

Exercise 2 (10 points). Prove that

\[ n \lfloor \log n \rfloor = \Theta(n \log n). \]

Exercise 3 (10 points). Prove or disprove: \( 2^n \log n = \Omega(e^n) \).

Exercise 4 (20 points). Show that for fixed \( k \), we have

\[ \binom{n}{k} = \frac{n^k}{k!} + O(n^{k-1}) \quad \text{and} \quad \binom{n+k}{k} = \frac{n^k}{k!} + O(n^{k-1}), \]

where \( \binom{n}{k} = n(n-1)(n-2) \cdots (n-k+1)/k! \) is the binomial coefficient.

Exercise 5 (20 points). Suppose that \( f \) and \( g \) are functions from the natural numbers to the positive real numbers. Suppose that the limit \( \lim_{n \to \infty} f(n)/g(n) \) exists and is positive. Prove or disprove that \( O(f) = O(g) \).

Exercise 6 (10 points). Suppose that it is known that each of the items in an array \( a[1..n] \) has one of two distinct values. Give a sorting algorithm for such arrays that takes time proportional to \( n \).

Exercise 7 (20 points). Assume that the running time of Mergesort is \( cn \log n + dn \), where \( c \) and \( d \) are machine-dependent constants. Show that if we implement the program on a particular machine and observe a running time \( t_n \) for some value of \( n \), then we can accurately estimate the running time for \( 2n \) by \( 2t_n(1 + 1/\log n) \), independent of the machine.