Teleportation
Andreas Klappenecker
The Problem

Alice wants to send the state of a quantum bit to Bob.

We assume that they share a pair of entangled quantum bits in the state $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$.

How can they do it if classical communication is allowed?
Let’s assume that Alice wants to teleport a quantum bit in the state $a|0\rangle + b|1\rangle$ to Bob and that they share a pair of entangled quantum bits such that the system is in the state: $(a|0\rangle+b|1\rangle) \otimes (|00\rangle+|11\rangle)/\sqrt{2}$. We claim that the following quantum circuit can solve the problem:

![Quantum Circuit Diagram]

The Quantum Circuit
State Evolution (1)

Initial state:

\[(a|0\rangle + b|1\rangle) \otimes (\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle).\]

Applying the controlled not yields

\[a|0\rangle \otimes (\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle) + b|1\rangle \otimes (\frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|01\rangle).\]
State Evolution (2)

\[ a|0\rangle \otimes \left( \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \right) + b|1\rangle \otimes \left( \frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|01\rangle \right). \]

Applying the Hadamard gate on the most significant qubit yields the state

\[ a\left( \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \right) \otimes \left( \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \right) + b\left( \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \right) \otimes \left( \frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|01\rangle \right). \]
Rewriting the State

Rewriting the state yields

\[
\begin{align*}
    a\left(\frac{1}{2}|000\rangle + \frac{1}{2}|011\rangle + \frac{1}{2}|100\rangle + \frac{1}{2}|111\rangle \right)
    &+ b\left(\frac{1}{2}|001\rangle + \frac{1}{2}|010\rangle - \frac{1}{2}|101\rangle - \frac{1}{2}|110\rangle \right).
\end{align*}
\]

or

\[
\frac{1}{2} \left( |00\rangle \otimes (a|0\rangle + b|1\rangle) + |01\rangle \otimes (a|1\rangle + b|0\rangle) \\
+ |10\rangle \otimes (a|0\rangle - b|1\rangle) + |11\rangle \otimes (a|1\rangle - b|0\rangle) \right).
\]
Measurement and Correction

\[
\frac{1}{2} \left( |00\rangle \otimes (a|0\rangle + b|1\rangle) + |01\rangle \otimes (a|1\rangle + b|0\rangle) + |10\rangle \otimes (a|0\rangle - b|1\rangle) + |11\rangle \otimes (a|1\rangle - b|0\rangle) \right).
\]

measuring the two most significant quantum bits yields

<table>
<thead>
<tr>
<th>Observation</th>
<th>Resulting State</th>
<th>Alice tells Bob</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>(</td>
<td>00\rangle \otimes (a</td>
</tr>
<tr>
<td>01</td>
<td>(</td>
<td>01\rangle \otimes (a</td>
</tr>
<tr>
<td>10</td>
<td>(</td>
<td>10\rangle \otimes (a</td>
</tr>
<tr>
<td>11</td>
<td>(</td>
<td>11\rangle \otimes (a</td>
</tr>
</tbody>
</table>