Honor-CSCE 222 Homework 3 (Due Oct. 17)

1 and 2. Prove, for each pair of expressions \((f(n), g(n))\) below, whether \(f(n)\) is \(O\), \(o\), \(\Omega\), \(\omega\) or \(\Theta\) of \(g(n)\). In each case, it is possible that more than one of these conditions is satisfied.

1. \(f(n) = n + (\log n)^2, g(n) = n + \log(n^2)\).
2. \(f(n) = e^n \ln n, g(n) = 2^n \log n\).

3. Compute the worst case time complexity of the following algorithm.

   for \(i = 1\) to \(n\) do
     for \(j = i\) to \(n^2\) do
       print \((i, j)\).

4. Prove by induction on \(n\) that \(\sum_{k=1}^{n} \frac{k + 2}{k(k + 1)2^{k+1}} = \frac{1}{2} - \frac{1}{(n + 1)2^{n+1}}\).

5. Prove by induction on \(n\) that \(\sum_{k=1}^{n} \frac{2^k}{k} \leq 2^n\).