1. Find a formula for $a_n$ that satisfies the recursive definition.

$$a_n = \begin{cases} 
0 & \text{when } n = 0 \\
(1 - a_{n-1})/2 & \text{when } n > 0.
\end{cases}$$

2. What is the number of functions $f$ from the set $\{1, 2, \ldots, 2n\}$ to the set $\{1, 2, \ldots, n\}$ so that $f(x) \leq \lceil x/2 \rceil$ for all $x$?

3. Prove that among $n + 2$ different integers from the set $\{-n, \ldots, -1, 0, 1, \ldots, n\}$, there are at least two integers that sum up to 0.

4. Prove that $\binom{4n}{2n} \binom{2n}{n} = \binom{4n}{n} \binom{3n}{n}$.

5. Evaluate $\sum_{k=0}^{n} \binom{2n + 1}{2k + 1}$.