1. Find a formula for \( a_n \) that satisfies the recursive definition.

\[
a_n = \begin{cases} 
1 & \text{when } n = 0 \\
a_{n-1}/2 + 1 & \text{when } n > 0.
\end{cases}
\]

2. What is the number of functions \( f \) from the set \( \{1, 2, \ldots, 2n\} \) to the set \( \{1, 2, \ldots, 2n\} \) so that \( f(x) \leq n \) if \( x \leq n \) and \( f(x) > n \) if \( x > n \)?

3. Prove that among any set of \( n + 1 \) different positive integers whose values are at most \( 2n \), there is always a pair of consecutive numbers.

4. Prove that \( P(n, r) = P(n - 1, r) + rP(n - 2, r - 1) + r(r - 1)P(n - 3, r - 2) + \cdots + r!P(n - r - 1, 0) \), where \( P(n, r) \) denotes the number of \( r \)-permutations on \( n \) elements.

5. Evaluate \( \sum_{k=1}^{n} \binom{n}{k} k^{2^{k-1}} \).